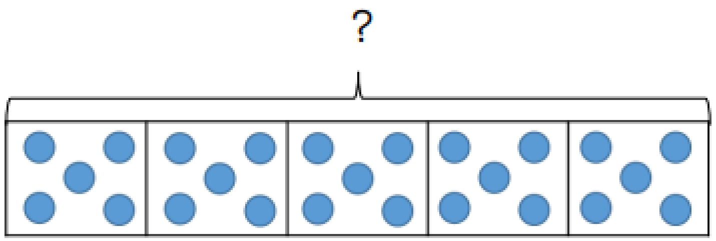
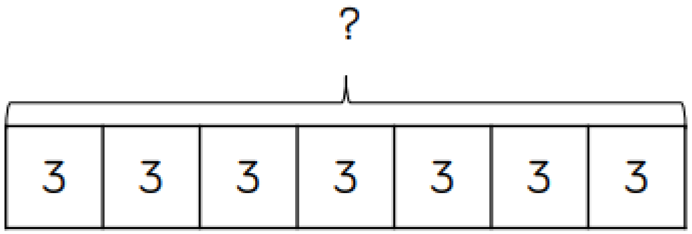
Multiplication & DIVISION

Bar Model

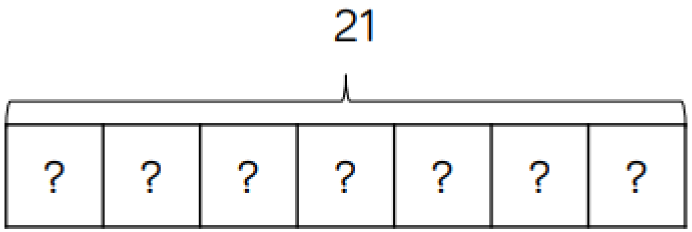


$5 \times 5 = 25$

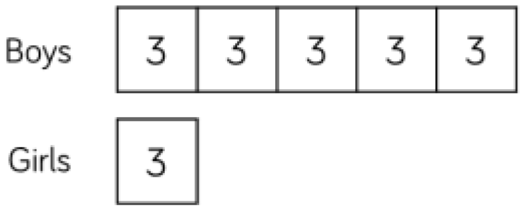


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups. It is important when solving word problems that the bar model represents the problem.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

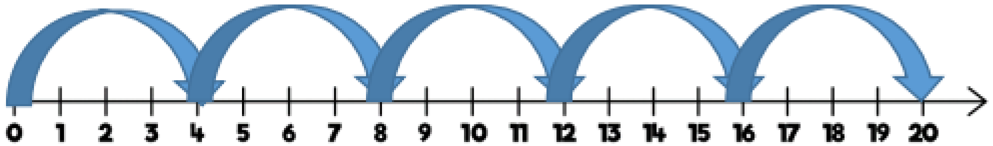
Benefits

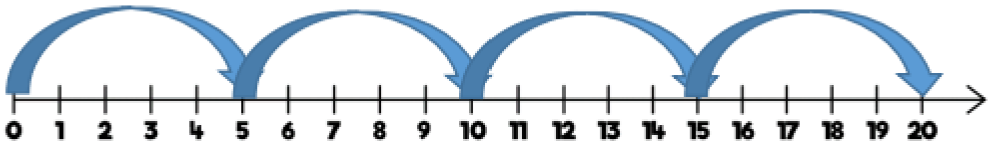
Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20. Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

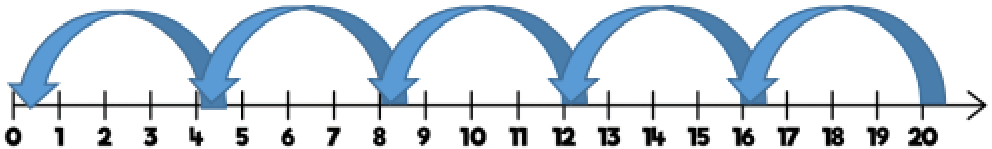
Number Lines (labelled)





$4 \times 5 = 20$

$5 \times 4 = 20$



$20 \div 4 = 5$

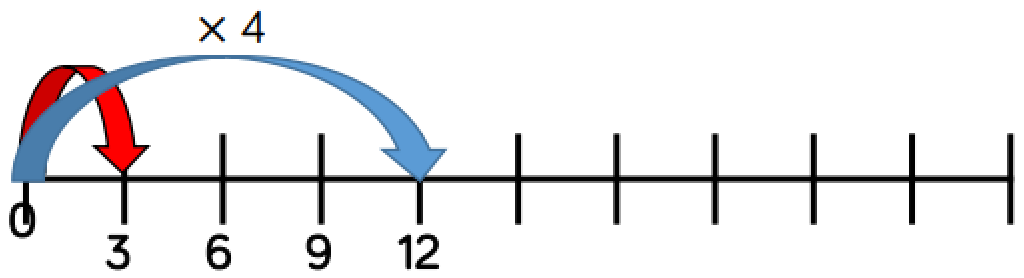
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

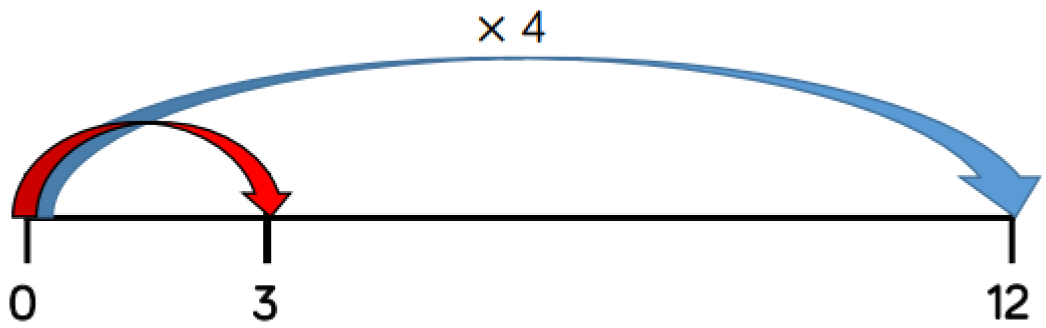
When multiplying, children start at 0 and then count on to find the product of the numbers. When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



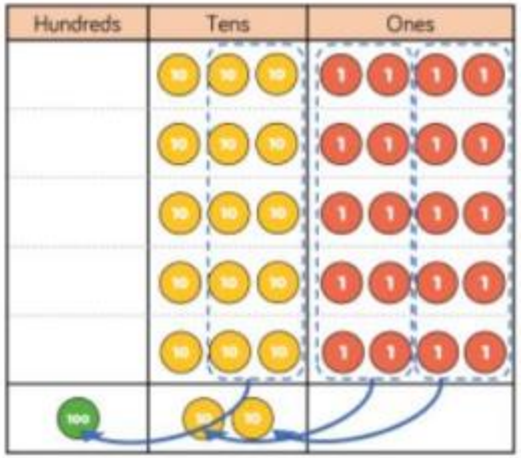
A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

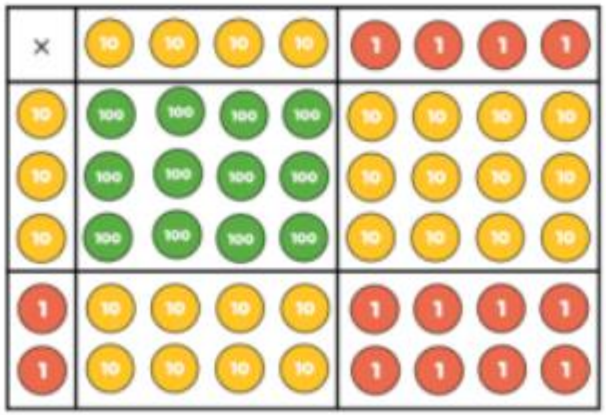
Children can use blank number lines to represent scaling as multiplication or division. Blank number lines with intervals can support children to represent scaling accurately.

Children can label intervals with multiples to calculate scaling problems. Blank number lines without intervals can also be used for children to represent scaling.

Place Value Counters (multiplication)



$$
\begin{array}{r}
34 \\
\times \quad 5 \\
\hline
170 \\
\hline
{\scriptstyle 1\ 2}
\end{array}
$$



$$
\begin{array}{r}
44 \\
\times\ 32 \\
\hline
8 \\
80 \\
120 \\
+\ 1200 \\
\hline
1408 \\
\hline
{\scriptstyle 1}
\end{array}
$$

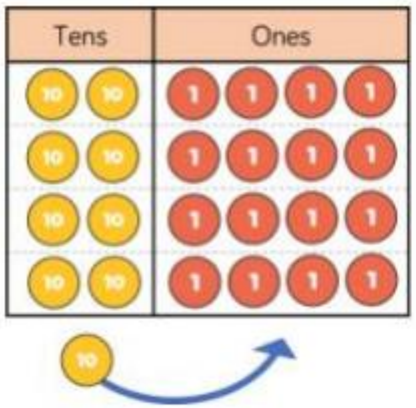
Benefits

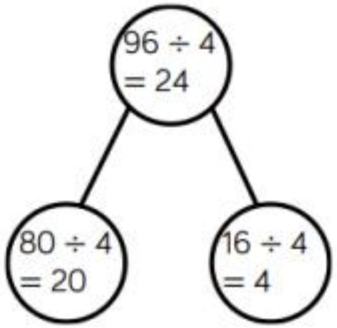
Using place value counters is an effective way to support children’s understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

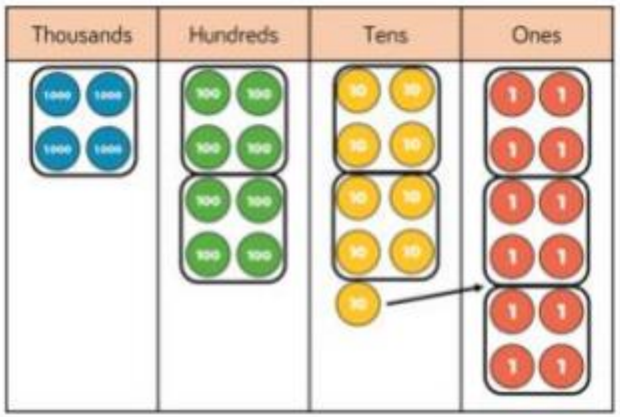
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2- digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)489^{1}2} \end{array}$$

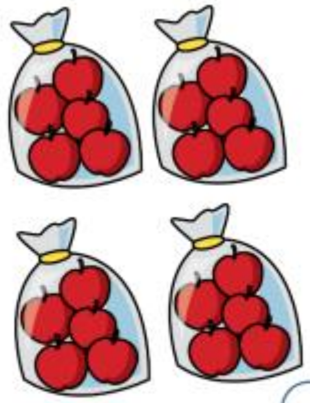
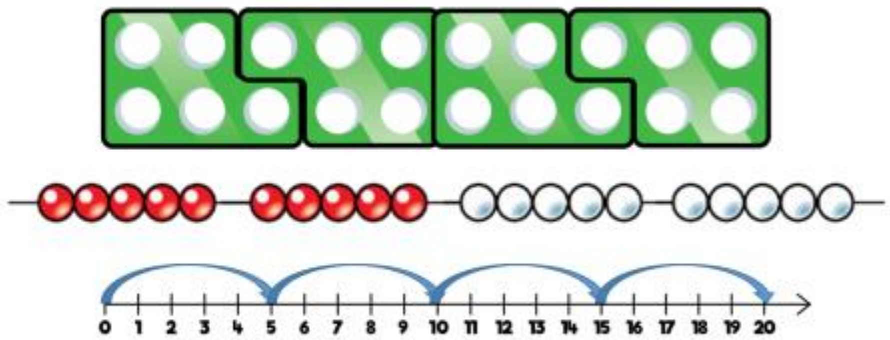
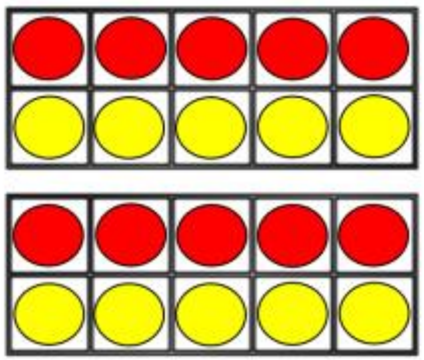
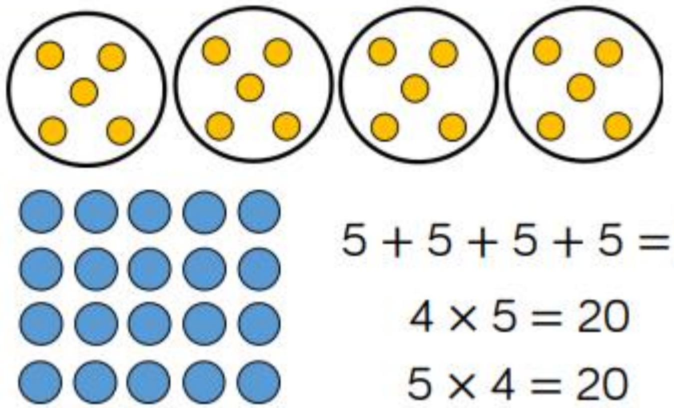
Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2- digit numbers by 2-digit numbers.

Multiplication

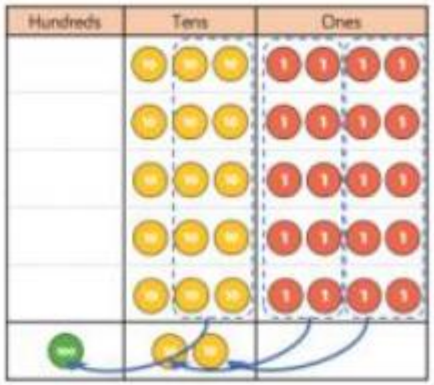
Skill: Solve 1-step problems using multiplication	Early Years/Year: 1/2
  One bag holds 5 apples. How many apples do 4 bags hold?   $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.

Skill: Multiply 2-digit numbers by 1-digit	Year: 3/4

	H	T	O		
		3	4		
×			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

$$34 \times 5 = 170$$

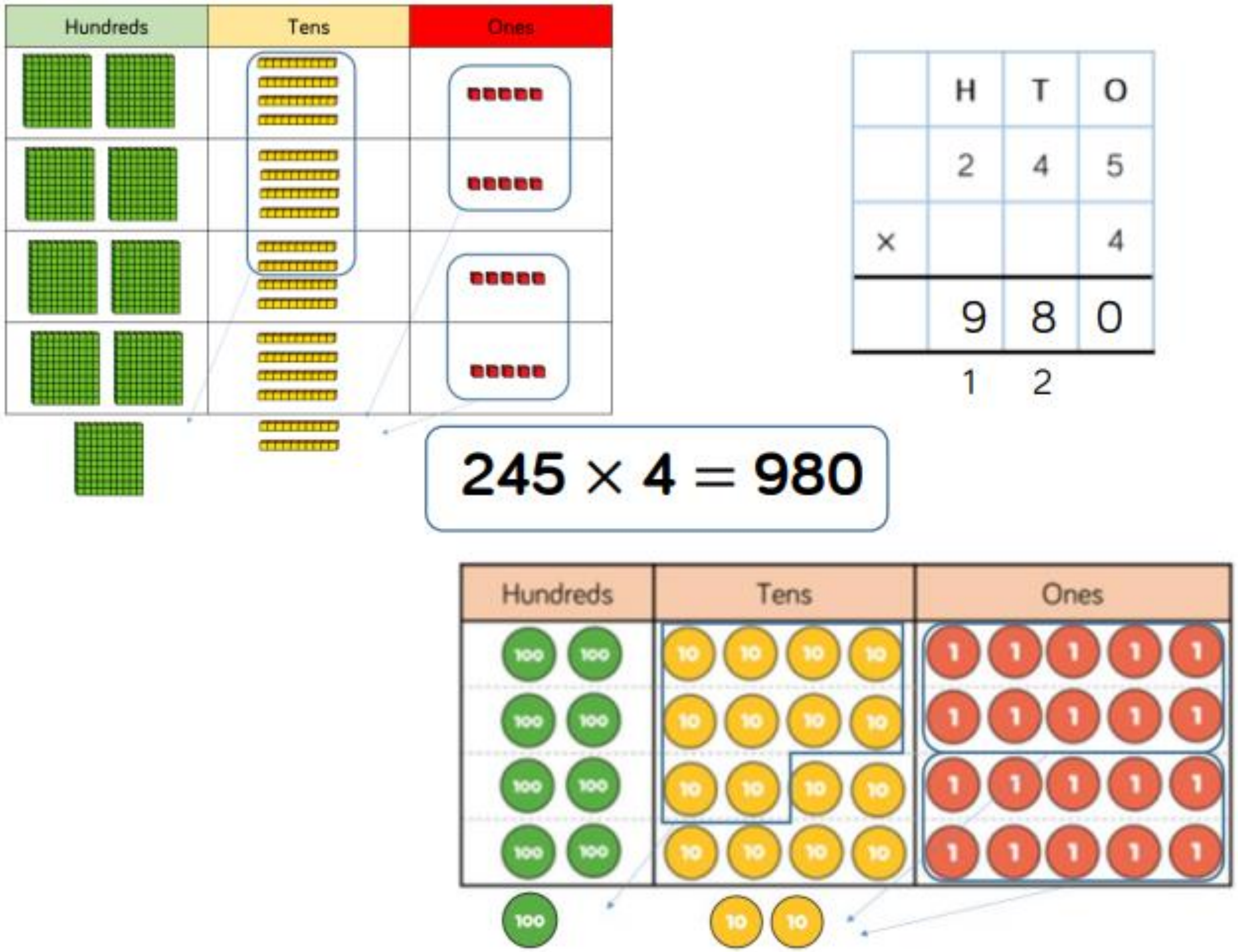
	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		



Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.

Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

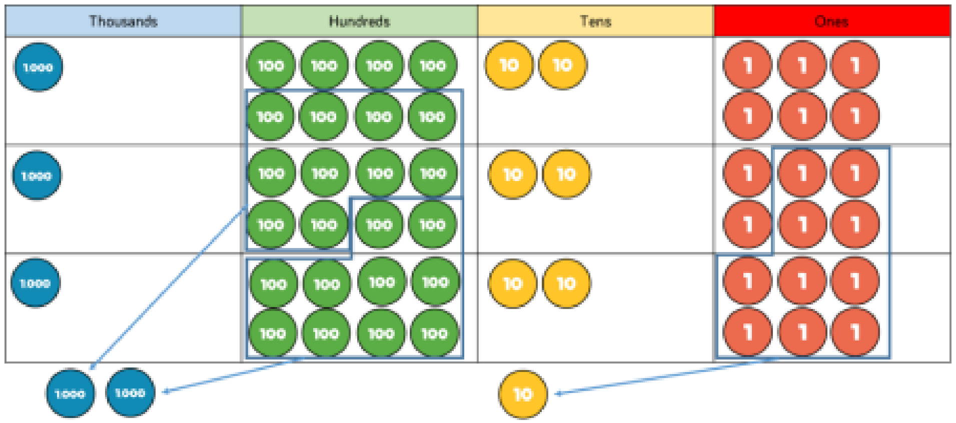
Skill: Multiply 3-digit numbers by 1-digit number | **Year: 4**



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method.

Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit	Year: 5

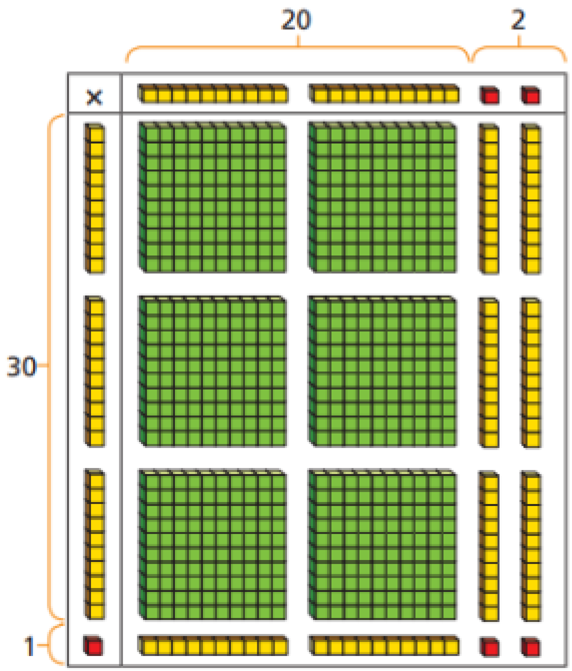


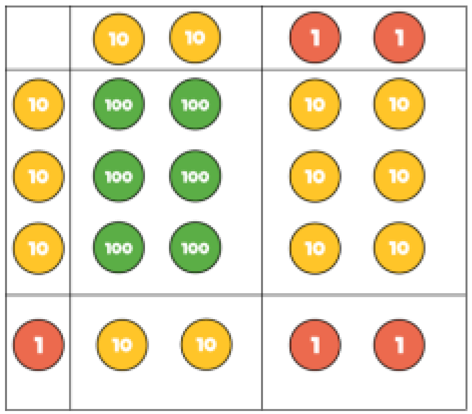
$$1{,}826 \times 3 = 5{,}478$$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit	Year: 5





×	20	2
30	600	60
1	20	2

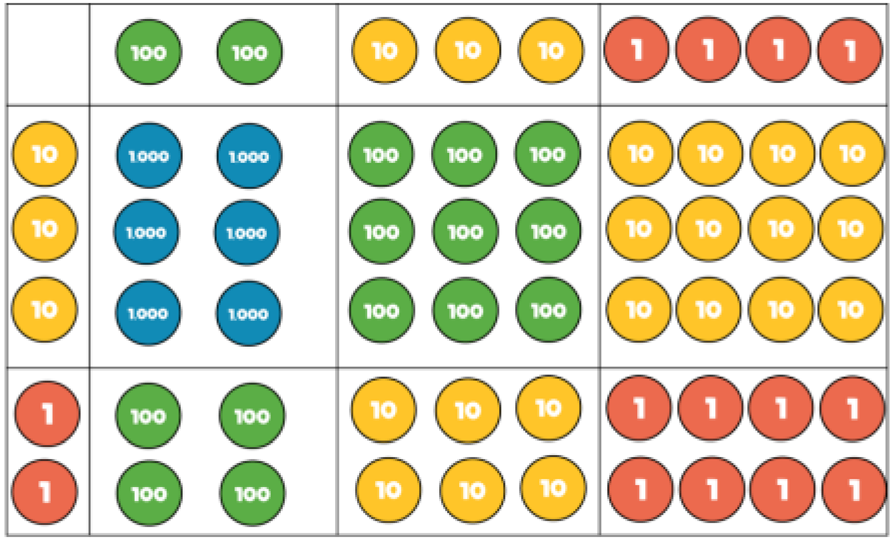
	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$22 \times 31 = 682$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.

The grid method matches the area model as an initial written method before moving on to the formal written multiplication

Skill: Multiply 3-digit numbers by 2-digit	Year: 5



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
1 7	1 0	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$234 \times 32 = 7{,}488$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Children should now move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit	Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2

1

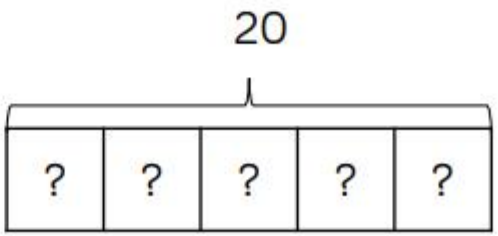
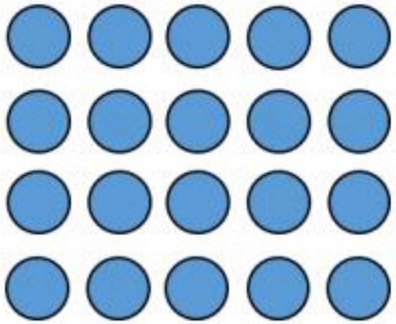
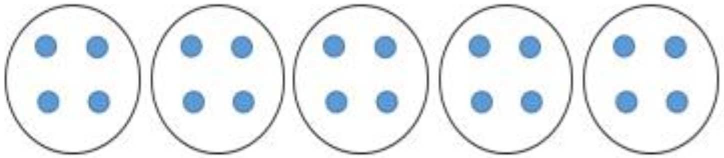
$$2{,}739 \times 28 = 76{,}692$$

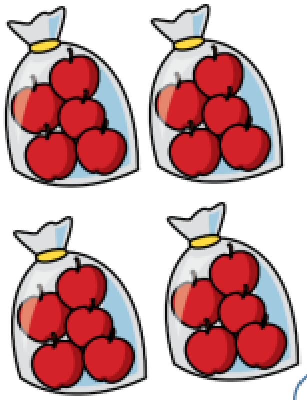
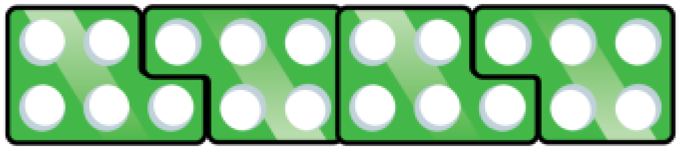
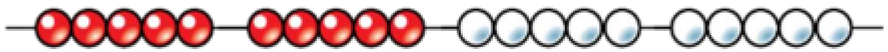
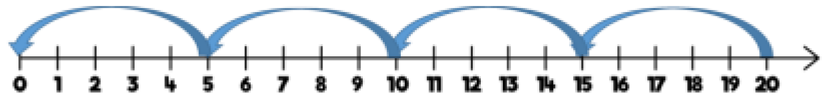
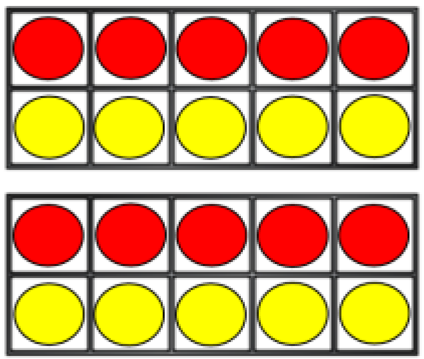
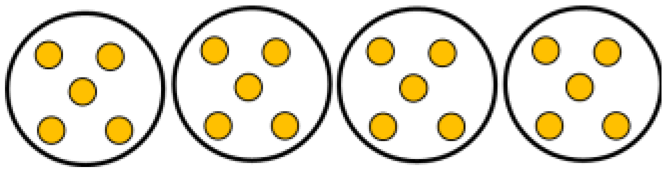
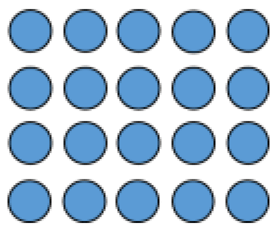
When multiplying 4-digits by 2-digits, children should be confident in using the formal written method.

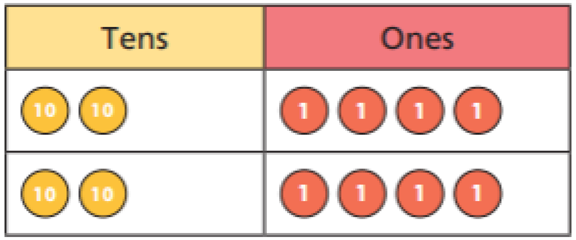
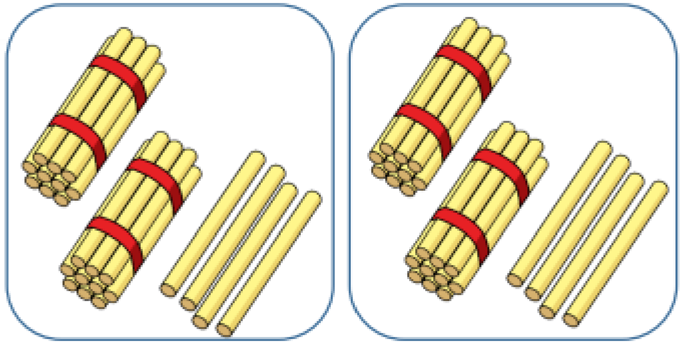
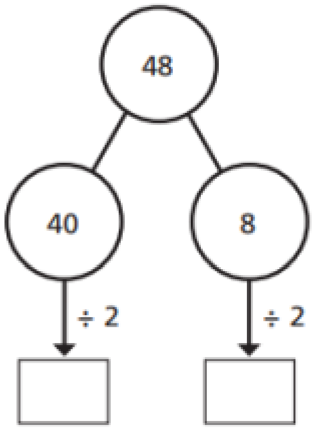
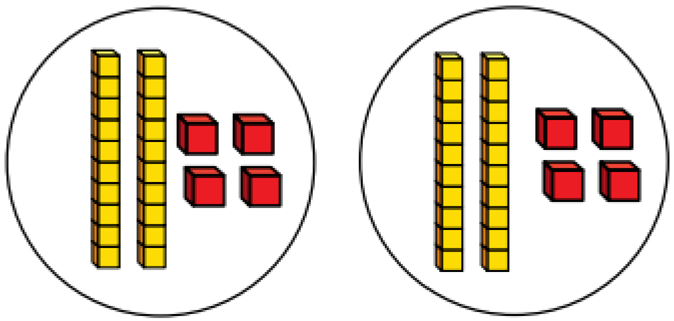
If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

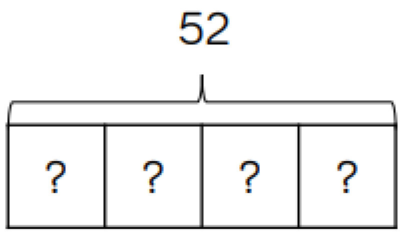
Division

Skill: Solve 1-step problems using multiplication	Early Years/Year: 1/2
  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?   $20 \div 5 = 4$	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.

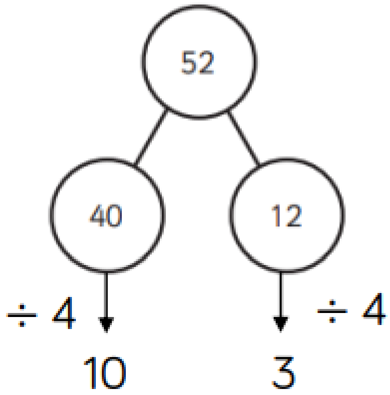
Skill: Solve 1-step problems using division	Year: 1/2
    There are 20 apples altogether. They are put in bags of 5. How many bags are there?    $20 \div 5 = 4$	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

Skill: Divide 2-digits by 1-digit (sharing with no exchange)	Year: 3
   $48 \div 2 = 24$ 	When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups. Part-whole models can provide children with a clear written method that matches the concrete representation.

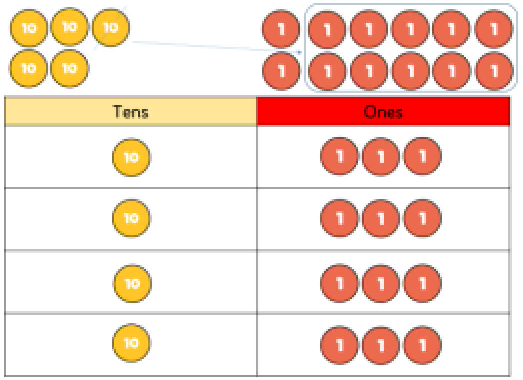
Skill: Divide 2-digits by 1-digit (sharing with exchange) | Year: 3/4



$$52 \div 4 = 13$$



$$10 + 3 = 13$$

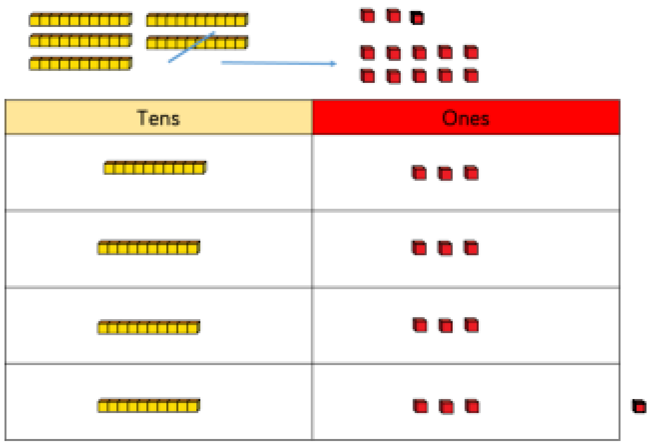


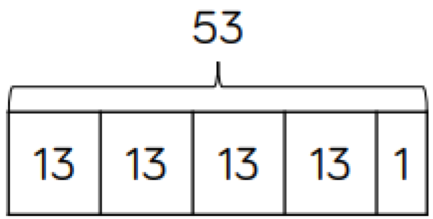
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.

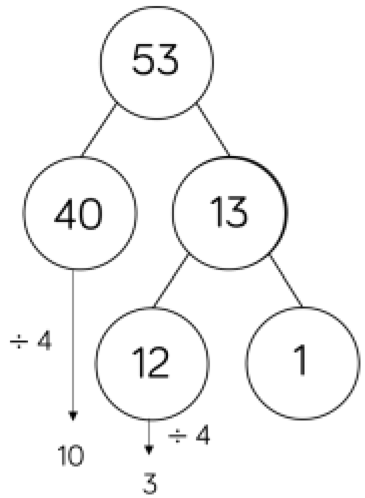
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

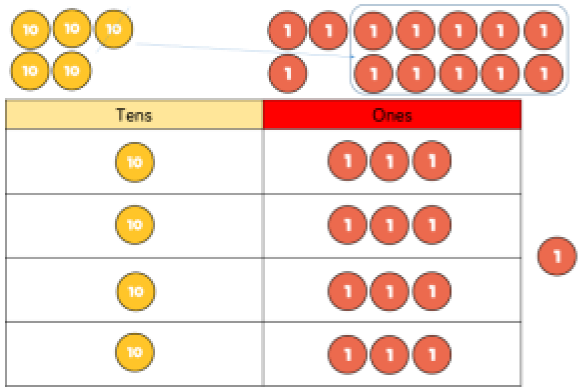
Skill: Divide 2-digits by 1-digit (sharing with remainders)	Year: 3/4







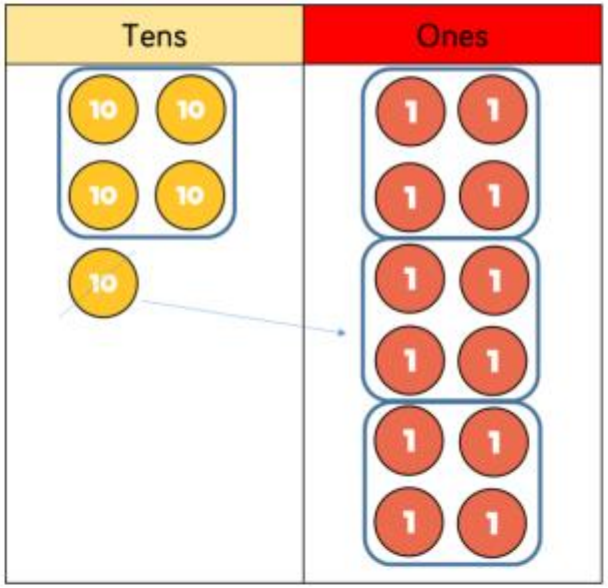
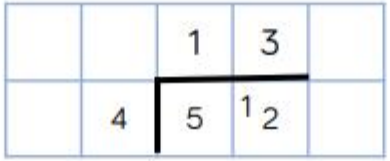
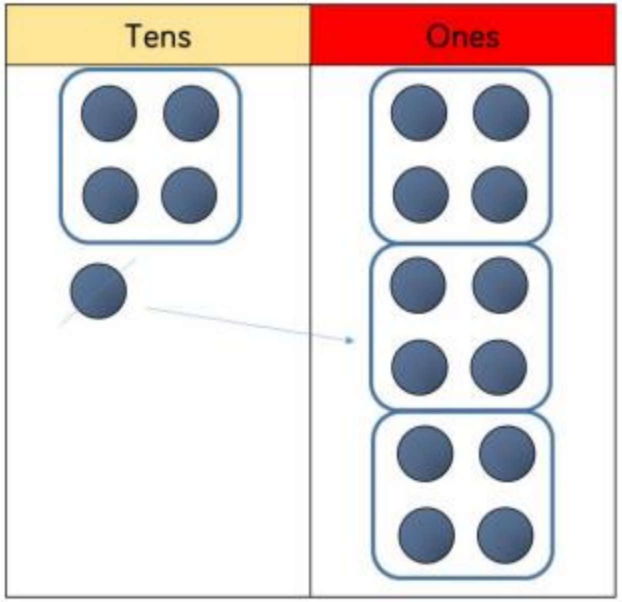
$$53 \div 4 = 13 \text{ r}1$$



When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.

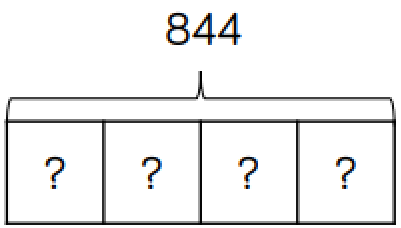
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.

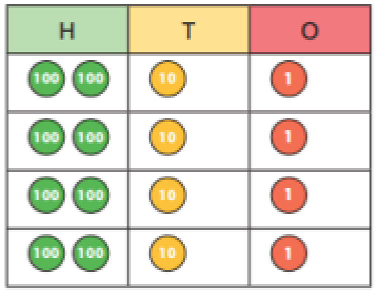
Flexible partitioning in a part-whole model supports this method.

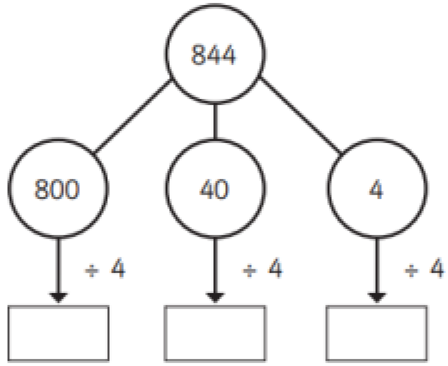
Skill: Divide 2-digits by 1-digit (grouping)	**Year: 4**
   $52 \div 4 = 13$	When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider ‘How many groups of 4 tens can we make?’ and ‘How many groups of 4 ones can we make?’ Remainders can also be seen as they are left ungrouped.

Skill: Divide 3-digits by 1-digit (sharing)	Year: 4

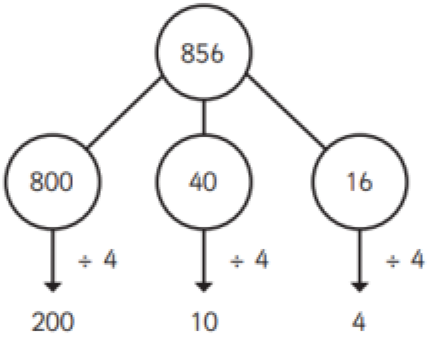
$844 \div 4 = 211$

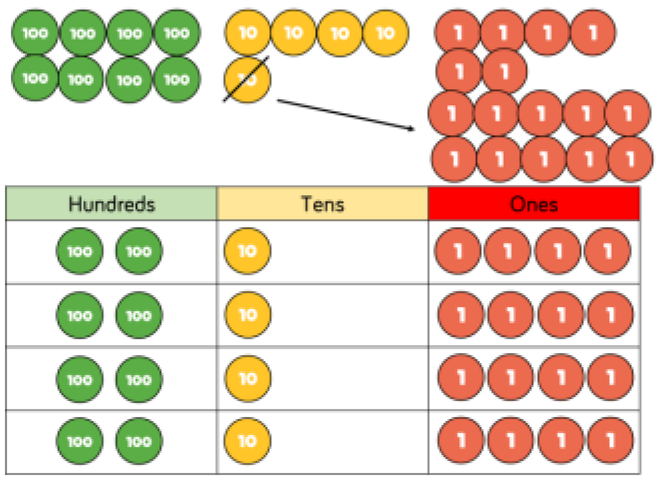






$856 \div 4 = 214$



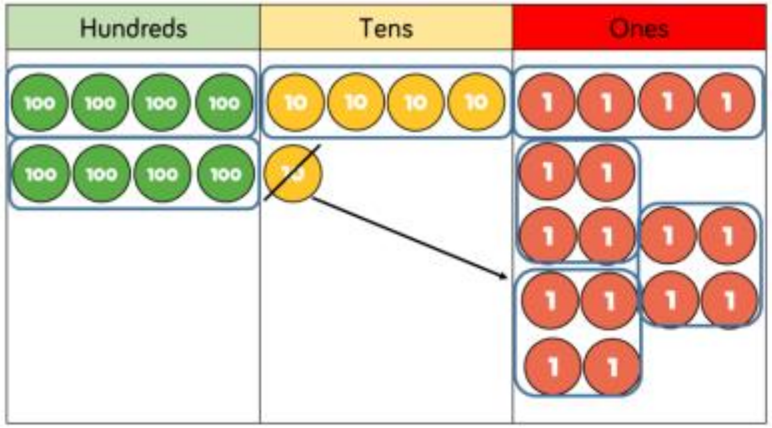


Children can continue to use place value counters to share 3- digit numbers into equal groups.

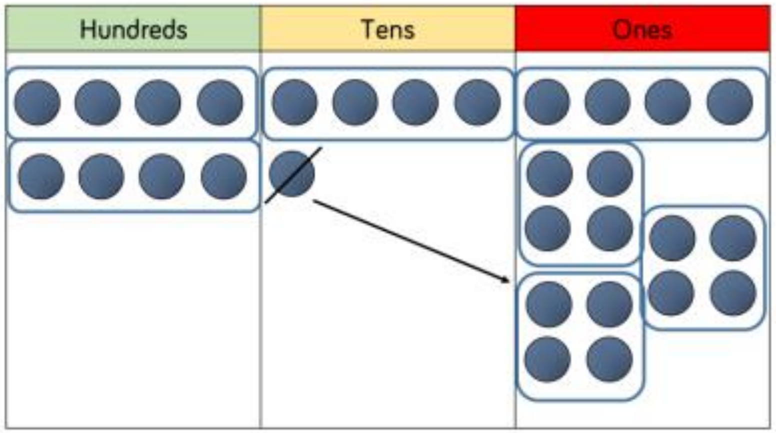
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.

This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.

Skill: Divide 3-digits by 1-digit (grouping)	Year: 5



		2	1	4
	4	8	5	$^{1}6$



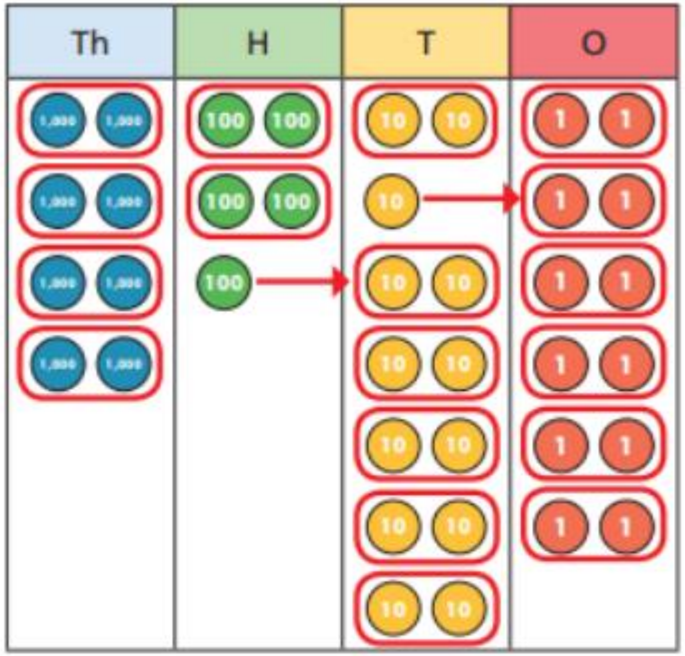
$$856 \div 4 = 214$$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding.

Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)	Year: 5



	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

$$8{,}532 \div 2 = 4{,}266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit.

Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Skill: Divide multi digits by 2-digits (short division)	Year: 6

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

When children begin to divide up to 4- digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long

Year: 6

		0	3	6	
1	2	4	3	2	
	−	3	6	0	(×30)
			7	2	
	−		7	2	(×6)
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9	
15	7	3	3	5	
−	6	0	0	0	(×40C
	1	3	3	5	
−	1	2	0	0	(×80)
		1	3	5	
−		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long	Year: 6

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

$$372 \div 15 = 24\frac{4}{5}$$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction.

This will depend on the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.